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Relating leptogenesis parameters to light neutrino masses

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Abstract: We obtain model independent relations among neutrino masses and leptogenesis parameters. We find exact relations that involve the CP asymmetries ϵ_{N_α}, the washout parameters $\tilde{m}_\alpha$ and $\theta_{\alpha\beta}$, and the neutrino masses m_i and M_α, as well as powerful inequalities that involve just $\tilde{m}_\alpha$ and m_i. We prove that the Yukawa interactions of at least two of the heavy singlet neutrinos are in the strong washout region ($\tilde{m}_\alpha \gg 10^{-3}$ eV).

Keywords: Baryogenesis, Neutrino Physics, Beyond Standard Model, Solar and Atmospheric Neutrinos.

Contents

1. Introduction

Singlet neutrinos with heavy Majorana masses and with Yukawa couplings to the active neutrinos generate light neutrino masses via the see-saw mechanism and a baryon asymmetry via leptogenesis [1], providing attractive qualitative solutions to these two important puzzles. To be quantitatively successful, the see-saw mechanism should lead to the two observed mass scales,

$$\begin{aligned} m_s &\equiv (\Delta m^2_{\rm sol})^{1/2} \sim 0.009\ eV, \\ m_a &\equiv (\Delta m^2_{\rm atm})^{1/2} \sim 0.05\ eV, \end{aligned} \tag{1.1}$$

while leptogenesis should lead to the value extracted from observations,

$$Y^{\rm obs}_{\mathcal{B}} \equiv \frac{n_B - n_{\overline{B}}}{s} = (8.7 \pm 0.3) \times 10^{-11}. \tag{1.2}$$

Unfortunately, because the leptogenesis parameters — the CP asymmetries and the washout factors — directly involve the heavy singlet neutrinos, we cannot realistically hope that they will be measured. In order to make further progress in the investigation of leptogenesis, it is highly desirable to relate the leptogenesis parameters to measurable mass parameters. The purpose of this work is to obtain such relations.

The relations that we obtain involve the washout parameters of all the heavy singlet neutrinos N_α. While most leptogenesis studies have focussed on the contributions from the decays of N_1, the lightest heavy singlet, it has been realized that, in general, the contributions from the decays of the heavier singlet neutrinos must not be neglected [2−4]. Indeed, our results reinforce this statement.

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2. Notations

The relevant Lagrangian terms involve, in addition to the N_α's, the light lepton SU(2)-doublets L_i and SU(2)-singlets E_i ($i = e, \mu, \tau$ is a flavor index), and the standard model Higgs H:

$$-\mathcal{L} = \frac{1}{2} M_\alpha N_\alpha N_\alpha + \lambda_{\alpha i} H N_\alpha L_i + Y_i H^\dagger L_i E_i. \tag{2.1}$$

Eq. (2.1) is written in the mass basis for the singlet neutrinos and for the charged leptons, that is, M and Y are diagonal.

The light neutrino mass matrix is given by

$$m_\nu = v^2 \lambda^T M^{-1} \lambda, \tag{2.2}$$

where $v = \langle H \rangle$. Reversing this relation, one can express the Yukawa couplings $\lambda_{\alpha i}$ in terms of the diagonal mass matrix M, the matrix $m = \mathrm{diag}(m_1, m_2, m_3)$ (where m_i^2 are the eigenvalues of $m_\nu m_\nu^\dagger$), the leptonic mixing matrix U and an orthogonal complex matrix R [5]:

$$\lambda = \frac{1}{v} M^{1/2}\ R\ m^{1/2}\ U^\dagger. \tag{2.3}$$

The baryon number generated from the decays of the N_α neutrinos can be written as follows:

$$Y_{\mathcal{B}} = -1.4 \times 10^{-3} \sum_{\alpha,\beta} \epsilon_{N_\alpha} \eta_{\alpha\beta}, \tag{2.4}$$

where ϵ_{N_α} is the CP asymmetry generated in N_α decays:

$$\epsilon_{N_\alpha} = \frac{\Gamma(N_\alpha \to \ell H) - \Gamma(N_\alpha \to \bar{\ell}\bar{H})}{\Gamma(N_\alpha \to \ell H) + \Gamma(N_\alpha \to \bar{\ell}\bar{H})}, \tag{2.5}$$

and $\eta_{\alpha\beta}$ denotes the efficiency factor related to the washout of the asymmetry ϵ_{N_α} due to N_β interactions. (If leptogenesis takes place at $T \lesssim 10^{12}$ GeV, flavor indices should be added [2, 6 – 8].) It is convenient for our purposes to further define a matrix of dimensionful quantities $\tilde{m}_{\alpha\beta}$:

$$\tilde{m} = v^2 M^{-1/2} \lambda\lambda^\dagger M^{-1/2}. \tag{2.6}$$

Note that $\tilde{m}$ is a positive matrix and, in particular, $|\tilde{m}_{\alpha\beta}|^2 \leq \tilde{m}_{\alpha\alpha}\tilde{m}_{\beta\beta}$. In terms of the parametrization (2.3), we have

$$\tilde{m}_{\alpha\beta} = \sum_i m_i R_{\alpha i} R^*_{\beta i}. \tag{2.7}$$

In a large part of the parameter space, the washout factors $\eta_{\alpha\alpha}$ depend on the mass and the couplings of N_α only via the combination $\tilde{m}_\alpha \equiv \tilde{m}_{\alpha\alpha}$ [9]. For example, for $M_1 \ll 10^{14}\ GeV$ and $\tilde{m}_\alpha \gg m_* = 2.2 \times 10^{-3}\ eV$, we have [10]

$$\eta_{\alpha\alpha} \approx \left(\frac{5.5 \times 10^{-4}\ eV}{\tilde{m}_\alpha} \right)^{1.16}. \tag{2.8}$$

When we talk in this work about the "washout parameters" we refer mainly to the $\tilde{m}_\alpha$'s. The off-diagonal terms in $\tilde{m}$ do, however, play important roles in leptogenesis. First, the CP asymmetries depend on $\mathcal{I}m(\tilde{m}_{\alpha\beta})$ (see, for example, eq. (4.9) below). Second, $|\tilde{m}_{\alpha\beta}|$ determines the overlap between the lepton doublet states ℓ_α and ℓ_β to which N_α and N_β decay, respectively [4]:

$$\begin{aligned} |\ell_\alpha\rangle &= (\lambda\lambda^\dagger)^{-1/2}_{\alpha\alpha} \sum_i \lambda_{\alpha i} |\ell_i\rangle, \\ \cos^2\theta_{\alpha\beta} &\equiv |\langle \ell_\alpha|\ell_\beta\rangle|^2 = |\tilde{m}_{\alpha\beta}|^2/(\tilde{m}_\alpha \tilde{m}_\beta). \end{aligned} \tag{2.9}$$

For the case of strong hierarchy between the masses and the lifetimes of, say, N_1 and N_2, and $\tilde{m}_1 \gg m_*$, the interactions of N_1 first project ϵ_{N_2} on the directions aligned with or orthogonal to ℓ_1 and then washout the asymmetry in the ℓ_1 direction [4]. (We assume here that the reheat temperature is higher than M_2.) For this case, we use an approximate expression for the total lepton asymmetry generated in N_2 and N_1 decays:

$$Y_\mathcal{B} \approx -1.4 \times 10^{-3} \left[\epsilon_{N_1}\eta_{11} + \epsilon_{N_2}\eta_{22}(\cos^2\theta_{12}\eta_{11} + \sin^2\theta_{12}) \right]. \tag{2.10}$$

3. The basic relations

The key point for our results is that $\tilde{m}^*\tilde{m}$ and $m_\nu m_\nu^\dagger$ are similar. In particular, the following three relations hold:

$$\begin{aligned} \det(m_\nu m_\nu^\dagger) &= \det(\tilde{m}^*\tilde{m}), \\ \mathrm{Sym}_2(m_\nu m_\nu^\dagger) &= \mathrm{Sym}_2(\tilde{m}^*\tilde{m}), \\ \mathrm{Tr}(m_\nu m_\nu^\dagger) &= \mathrm{Tr}(\tilde{m}^*\tilde{m}), \end{aligned} \tag{3.1}$$

where $\mathrm{Sym}_2(A) = \frac{1}{2}\left\{ [\mathrm{Tr}(A)]^2 - \mathrm{Tr}(A^2) \right\}$.

These equations can be written as exact relations involving the light neutrino masses m_i, the washout parameters $\tilde{m}_\alpha$, and the off-diagonal terms $\tilde{m}_{\alpha\beta}$. The latter can be expressed in terms of the CP asymmetries ϵ_{N_α} and the projections $\cos^2\theta_{\alpha\beta}$.

These equalities [as well as the explicit form (2.7)] can be further used to obtain simple inequalities involving only the washout parameters $\tilde{m}_\alpha$ and the light neutrino masses m_i. (Some of these inequalities have been previously derived in ref. [11].) In particular, we are able to show that some (and in some cases all) of the N_α interactions are in the strong washout region.

We note that there is no additional information for us in the leptonic mixing angles. The reason is that $\tilde{m}$ is independent of the mixing angles. This can be seen by noting that $\lambda\lambda^\dagger$ is independent of U, eq. (2.3), or directly from eq. (2.7).

If leptogenesis takes place at $T \lesssim 10^{12}$ GeV, then light-flavor effects are important. The individual light-flavor asymmetries $\epsilon^i_{N_\alpha}$ and washout parameters $\tilde{m}^i_\alpha$ play separate role in the time evolution of the lepton asymmetry. Our results are, however, still relevant, because they constrain the sums of the flavored parameters:

$$\sum_i \epsilon^i_{N_\alpha} = \epsilon_{N_\alpha}, \quad \sum_i \tilde{m}^i_\alpha = \tilde{m}_\alpha. \tag{3.2}$$

Note that the $\epsilon^i_{N_\alpha}$-asymmetries could carry either sign and, in particular, there could be cancellations in their sum. On the other hand, the $\tilde{m}^i_\alpha$-parameters are all positive, which makes constraints on their sum more significant.

We apply eqs. (3.1) to two cases, differing in the number of singlet neutrinos N_α that are added to the SM. In the "3+2" framework, two such neutrinos are assumed to be relevant to the see-saw mechanism and to leptogenesis, while in the "3+3" framework, there are three. The $3+2$ case is actually a special limit of the $3+3$ framework. When one of the three $\tilde{m}_\alpha \to 0$ (that is, $(\lambda\lambda^\dagger)_{\alpha\alpha} \to 0$ and/or $M_\alpha \to \infty$), the corresponding N_α becomes irrelevant to both the see-saw mechanism and leptogenesis, and the model reduces to effectively a $3+2$ model. We checked that all our results in the $3+3$ framework indeed reproduce the $3+2$ results in this limit.

We do not explicitly consider $3+m$ models with $m > 3$ because, in general, they are similar to the $3+3$ model. To understand that, note that ℓ_2 (ℓ_3) has, in general, a component that is orthogonal to ℓ_1 (ℓ_1 and ℓ_2). Consequently, part of the asymmetries generated by the decays of N_2 and N_3 is protected against washout [4]. The light flavor space is, however, three dimensional and therefore spanned, in general, by ℓ_1, ℓ_2 and ℓ_3. Consequently, there is no component in $\ell_{\alpha>3}$ that is orthogonal to all three, and the asymmetries $\epsilon_{N_{\alpha>3}}$ are expected to be washed out. Yet, the useful inequality (6.2) that we derive in the $3+3$ framework generalizes to the $3+m$ framework.

4. The 3 + 2 framework

When we have only two singlet neutrinos [$\alpha = 1,2$ in eq. (2.1)], one of the light mass eigenvalues vanishes and the other two mass eigenvalues are fixed by phenomenology to one of two discrete possibilities, either normal hierarchy (NH) or inverted hierarchy (IH):

$$\begin{aligned} m_1 &= 0, \qquad m_2 = m_s, \qquad\qquad m_3 = m_a \qquad \text{(NH)}, \\ m_1 &= 0, \qquad m_{2,3} \approx m_a, \qquad m_3 - m_2 = m_s^2/(2m_a) \qquad \text{(IH)}. \end{aligned} \tag{4.1}$$

Using eqs. (3.1), we have

$$m_2 m_3 = \tilde{m}_1 \tilde{m}_2 - |\tilde{m}_{12}|^2, \tag{4.2}$$

$$m_2^2 + m_3^2 = \tilde{m}_1^2 + \tilde{m}_2^2 + 2\mathcal{R}e(\tilde{m}_{12}^2). \tag{4.3}$$

Using eqs. (4.2) and (4.3), we obtain two inequalities:

$$\tilde{m}_2 + \tilde{m}_1 \geq m_3 + m_2, \tag{4.4}$$

$$|\tilde{m}_2 - \tilde{m}_1| \leq m_3 - m_2. \tag{4.5}$$

These inequalities have been previously derived in ref. [11]. We derived various additional inequalities:

$$\tilde{m}_{1,2} \geq m_2, \tag{4.6}$$

$$\tilde{m}_1 \tilde{m}_2 \geq m_2 m_3, \tag{4.7}$$

$$m_2/m_3 \leq \tilde{m}_1/\tilde{m}_2 \leq m_3/m_2. \tag{4.8}$$

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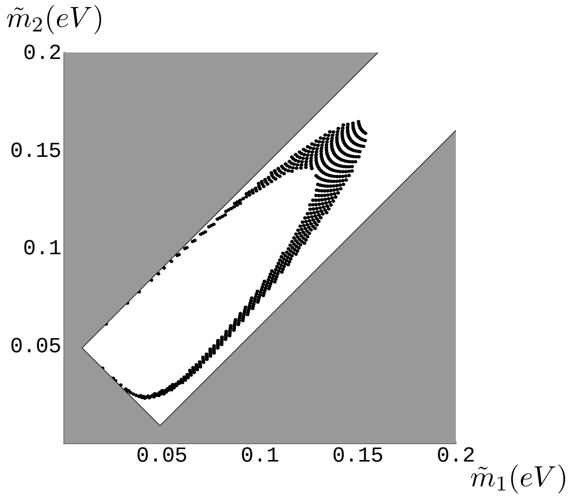

Figure 1: The constraints in the $\tilde{m}_1 - \tilde{m}_2$ plane in the $3+2$ case with normal hierarchy. The grey region is forbidden by eqs. (4.4) and (4.5). The black region is derived by scanning the parameter space (fixing $M_1 = 10^{12}\ GeV$ and $M_2/M_1 = 10$) and requiring that the resulting baryon asymmetry would be within the 3σ range [eq. (1.2)].

(The inequality $\tilde{m}_1 \geq m_2$ was derived in ref. [12].) However, eqs. (4.6)–(4.8) are redundant when the constraints (4.4) and (4.5) are imposed.

The combination of the inequalities (4.4) and (4.5), together with the known values of m_2 and m_3 [eq. (4.1)], constrains the allowed region in the $\tilde{m}_1 - \tilde{m}_2$ plane in a very significant way. We plot these constraints in figure 1. In particular, we can draw the following conclusions:

(i) For NH, both $\tilde{m}_\alpha$ are above m_s [11] and at least one of them is above $m_a/2$. The two washout factors are within a factor of $m_a/m_s \sim 6$ of each other.

(ii) For IH, both $\tilde{m}_\alpha$ are above m_a, while the difference between them is very small, $\leq m_s^2/(2m_a)$.

(iii) In either case, both N_2 interactions and N_1 interactions are in the strong washout regime.

As concerns the two CP asymmetries, $\epsilon_{N_{1,2}}$, they can be written as ($x_{12} \equiv M_1/M_2$)

$$\epsilon_{N_\alpha} = f_\alpha(x_{12}) \frac{M_\alpha \mathcal{I}m(\tilde{m}_{12}^2)}{v^2 \tilde{m}_\alpha}. \tag{4.9}$$

The functions $f_\alpha(x_{12})$ can be found in the literature [13]. Instead of the inequalities (4.4) and (4.5), one can combine eqs. (4.2), (4.3) and (4.9) to obtain an *exact* relation between the neutrino masses, the washout parameters and the CP asymmetries [11]:

$$4(\tilde{m}_1\tilde{m}_2 - m_2 m_3)^2 - (\tilde{m}_1^2 + \tilde{m}_2^2 - m_2^2 - m_3)^2 = 4\frac{v^4 \tilde{m}_1^2 \epsilon_{N_1}^2}{M_1^2 [f_1(x_{12})]^2} = 4\frac{v^4 \tilde{m}_2^2 \epsilon_{N_2}^2}{M_2^2 [f_2(x_{12})]^2}. \tag{4.10}$$

5. The $3+2$ framework with strong M_2/M_1 hierarchy

While our main focus here is on model independent relations, we can gain some further understanding by assuming mass hierarchy between the two singlet neutrinos. (It also explains features of the black region in figure 1 which corresponds to $M_2/M_1 = 10$.) In the hierarchical case ($x_{12} \ll 1$), we have

$$\begin{aligned} f_1(x_{12}) &= -3/(16\pi), \\ f_2(x_{12}) &= -x_{12}^2[\ln(x_{12})+1]/(4\pi), \\ \frac{\epsilon_{N_2}}{\epsilon_{N_1}} &= -\frac{4}{3}\frac{\tilde{m}_1}{\tilde{m}_2}x_{12}\left(\ln\frac{1}{x_{12}}-1\right). \end{aligned} \tag{5.1}$$

Using eqs. (2.4) and (2.8), we can give a rough estimate of the ratio between the respective contributions to $Y_{\mathcal{B}}$:

$$\frac{|\epsilon_{N_2}|/\tilde{m}_2}{|\epsilon_{N_1}|/\tilde{m}_1} \sim \frac{\tilde{m}_1^2}{\tilde{m}_2^2}\frac{M_1}{M_2}\left(\ln\frac{M_2}{M_1}-1\right). \tag{5.2}$$

We would like to emphasize the following points:

(i) For a mild hierarchy between M_1 and M_2, N_2-leptogenesis must not be neglected. (In this case, η_{12} and η_{21} have to be taken into account.)

(ii) In the NH case, only for a very strong hierarchy, $M_2/M_1 \gg 10^2$, it is guaranteed that N_1 leptogenesis dominates. For IH, the contribution from ϵ_{N_1} is always larger.

(iii) Given that ϵ_{N_2} and ϵ_{N_1} have opposite signs, partial (and potentially significant) cancellation between the two contributions is quite possible.

In the $3+3$ framework, when the heavy neutrino masses are strongly hierarchical, ϵ_{N_1} is subject to an upper bound [14, 15]:

$$|\epsilon_{N_1}| \leq \epsilon^{\rm DI} \equiv \frac{3}{16\pi}\frac{M_1(m_3-m_2)}{v^2}. \tag{5.3}$$

We first note that an even stronger bound applies to $|\epsilon_{N_2}|$:

$$|\epsilon_{N_2}| \leq \frac{4}{3}\frac{M_1}{M_2}\left[\ln\left(\frac{M_2}{M_1}\right)-1\right]\epsilon^{\rm DI}. \tag{5.4}$$

Second, we note that $m_3 - m_2$ is fixed to either $m_a - m_s$ (NH) or $m_s^2/(2m_a)$ (IH) [16], and so we can be more specific in eq. (5.3):

$$\epsilon^{\rm DI} = \begin{cases} M_1/(2.5\times 10^{16}\ GeV) & \text{NH} \\ M_1/(10^{18}\ GeV) & \text{IH} \end{cases} \tag{5.5}$$

In addition, since for NH(IH) $\tilde{m}_1 \geq m_s(m_a)$, we have $\eta_{11} \leq 0.04(0.006)$. The upper bounds on ϵ_{N_1} and on η_{11} give lower bounds on M_1,

$$M_1 \gtrsim \begin{cases} 3.6\times 10^{10}\ GeV & \text{NH} \\ 1.3\times 10^{13}\ GeV & \text{IH} \end{cases} \tag{5.6}$$

Alternatively, one can write upper bounds on $\tilde{m}_1$:

$$\tilde{m}_1 \lesssim \begin{cases} m_s \times [M_1/(3.6 \times 10^{10}\ GeV)]^{0.86} & \text{NH} \\ m_a \times [M_1/(1.3 \times 10^{13}\ GeV)]^{0.86} & \text{IH} \end{cases} \tag{5.7}$$

6. The 3 + 3 framework

Within the 3 + 3 framework, we can distinguish three different types of light neutrino spectra, Normal hierarchy (NH), inverted hierarchy (IH), and quasi degeneracy (QD):

$$\begin{aligned} &m_1 \ll m_s, && m_2 \approx m_s, && m_3 \approx m_a && \text{(NH)};\\ &m_1 \ll m_a, && m_{2,3} \approx m_a, && m_3 - m_2 = \frac{m_s^2}{2m_a} && \text{(IH)};\\ &m_i \approx \bar{m} \gg m_a, && m_3 - m_2 = \frac{m_a^2}{2\bar{m}}, && m_2 - m_1 = \frac{m_s^2}{2\bar{m}} && \text{(QD)}. \end{aligned}$$

Using eq. (2.7), we obtain lower bounds on the washout parameters (the first relation was derived in ref. [17]):

$$\tilde{m}_\alpha \geq m_1, \tag{6.1}$$

$$\tilde{m}_1 + \tilde{m}_2 + \tilde{m}_3 \geq m_1 + m_2 + m_3. \tag{6.2}$$

It is interesting to note that the inequality (6.2) can be easily generalized to any number $m \geq 3$ of singlet neutrinos. Using eq. (2.7) and the orthonormality of the $3 \times m$ matrix R, $R^T R = 1_{3\times 3}$, it is straightforward to prove that $\sum_{\alpha=1}^m \tilde{m}_\alpha \geq \sum_{i=1}^3 m_i$.

Evaluating eqs. (3.1), we obtain

$$m_1^2 + m_2^2 + m_3^2 = \tilde{m}_1^2 + \tilde{m}_2^2 + \tilde{m}_3^2 + 2\mathcal{R}e(\tilde{m}_{12}^2 + \tilde{m}_{23}^2 + \tilde{m}_{13}^2), \tag{6.3}$$

$$\begin{aligned} m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2 = & (\tilde{m}_{11}\tilde{m}_{22} - |\tilde{m}_{12}|^2)^2 \\ & + (\tilde{m}_{11}\tilde{m}_{33} - |\tilde{m}_{13}|^2)^2 + (\tilde{m}_{22}\tilde{m}_{33} - |\tilde{m}_{23}|^2)^2 \\ & + 2\mathcal{R}e\left[(\tilde{m}_{11}\tilde{m}_{23}^* - \tilde{m}_{12}\tilde{m}_{31})^2\right. \\ & \left. + (\tilde{m}_{22}\tilde{m}_{31}^* - \tilde{m}_{23}\tilde{m}_{12})^2 + (\tilde{m}_{33}\tilde{m}_{12}^* - \tilde{m}_{31}\tilde{m}_{23})^2\right]. \end{aligned} \tag{6.4}$$

$$\begin{aligned} m_1 m_2 m_3 = & \tilde{m}_{11}\tilde{m}_{22}\tilde{m}_{33} - \tilde{m}_{33}|\tilde{m}_{12}|^2 - \tilde{m}_{22}|\tilde{m}_{13}|^2 \\ & - \tilde{m}_{11}|\tilde{m}_{23}|^2 + 2\mathcal{R}e(\tilde{m}_{12}\tilde{m}_{23}\tilde{m}_{31}) \\ \leq & \tilde{m}_1\tilde{m}_2\tilde{m}_3. \end{aligned} \tag{6.5}$$

Using the fact that $\tilde{m}$ is hermitian and positive, and the general property that for any positive definite matrix A one has $\mathrm{Tr}[(AA^\dagger)^{-1/2}] \geq \mathrm{Tr}[(AA^*)^{-1/2}]$ [18], we obtain

$$\tilde{m}_1\tilde{m}_2 + \tilde{m}_2\tilde{m}_3 + \tilde{m}_3\tilde{m}_1 - (m_1 m_2 + m_2 m_3 + m_3 m_1) \geq |\tilde{m}_{12}|^2 + |\tilde{m}_{23}|^2 + |\tilde{m}_{13}|^2. \tag{6.6}$$

Eqs. (6.3) and (6.6) can be combined to give

$$\tilde{m}_1^2 + \tilde{m}_2^2 + \tilde{m}_3^2 - 2(\tilde{m}_1\tilde{m}_2 + \tilde{m}_1\tilde{m}_3 + \tilde{m}_2\tilde{m}_3) \leq m_1^2 + m_2^2 + m_3^2 - 2(m_1 m_2 + m_1 m_3 + m_2 m_3). \tag{6.7}$$

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We now use the above equations and inequalities to obtain lower bounds on the $\tilde{m}_\alpha$'s. We denote by $\tilde{m}_a, \tilde{m}_b, \tilde{m}_c$ the smallest, intermediate and largest $\tilde{m}_\alpha$, respectively. The left hand side of the inequality (6.7) is minimized for $\tilde{m}_b - \tilde{m}_a = 0$. It is maximized (for $\tilde{m}_c \geq 2(\tilde{m}_b + \tilde{m}_a)$) by maximal $\tilde{m}_c$ which, according to eq. (6.2), is given by $\sum_i m_i - (\tilde{m}_b + \tilde{m}_a)$. We can then write an inequality that depends on the sum of the two smallest $\tilde{m}_\alpha$:

$$3(\tilde{m}_a + \tilde{m}_b)^2 - 4\sum_i m_i(\tilde{m}_a + \tilde{m}_b) + 4\sum_{i<j} m_i m_j \leq 0, \tag{6.8}$$

This inequality leads to an interesting lower bound,

$$\tilde{m}_a + \tilde{m}_b \geq \frac{2}{3}\sum_i m_i - \frac{2}{3}\left(\sum_i m_i^2 - \sum_{i<j} m_i m_j\right)^{1/2}. \tag{6.9}$$

This bound has interesting implications for models of N_2 leptogenesis that are based on $\tilde{m}_1$ in the weak washout region [19–21], where it gives $\tilde{m}_2 \gtrsim m_s$. (A qualitative statement in this regard was made in ref. [20].)

Eqs. (6.1), (6.2) and (6.9) lead to the following lower bounds:

- Normal hierarchy:

$$\tilde{m}_c \geq \frac{m_a}{3}\left(1 + \frac{m_s}{m_a}\right), \quad \tilde{m}_b \geq \frac{m_s}{2}\left(1 - \frac{m_s}{4m_a}\right), \quad \tilde{m}_a \geq m_1. \tag{6.10}$$

- Inverted hierarchy:

$$\tilde{m}_c \geq \frac{2m_a}{3}, \quad \tilde{m}_b \geq \frac{m_a}{2}, \quad \tilde{m}_a \geq m_1. \tag{6.11}$$

- Quasi degeneracy:

$$\tilde{m}_\alpha \geq \bar{m}. \tag{6.12}$$

We conclude that, for hierarchical (quasi-degenerate) light neutrino masses, at least two (all three) of the $\tilde{m}_\alpha$ are in the strong washout region.

7. Conclusions

We investigated the relations between leptogenesis parameters and light neutrino masses. In particular, we derived exact relations between elements of the $\tilde{m}$ matrix [defined in eq. (2.6)], relevant to leptogenesis, and the light neutrino masses. The diagonal elements, $\tilde{m}_\alpha$, determine the $\Delta L = 1$ washout effects. As concerns the off-diagonal ones, $\mathcal{I}m(\tilde{m}_{\alpha\beta})$ determine the size of the CP asymmetries, while $|\tilde{m}_{\alpha\beta}|$ is related to projections (in heavy flavor space) of the asymmetries generated by heavy singlet neutrinos due to interactions of lighter singlets.

The resulting equations lead to interesting exact relations, such as eq. (4.10), between the washout parameters, CP asymmetries and neutrino masses. The various relations lead to simple inequalities between the washout parameters $\tilde{m}_\alpha$ and the light neutrino masses m_i, see eqs. (4.4)–(4.8) for the $3+2$ framework and (6.1)–(6.9) for the $3+3$ framework.

For light neutrino masses with normal hierarchy, we find the following results:

- In the 3+2 framework, both N_1 and N_2 interactions are in the strong washout region, with both $\tilde{m}_\alpha \geq 0.009$ eV and at least one ≥ 0.025 eV.
- In the $3+3$ framework, at least two N_α's have interactions in the strong washout region, with $\tilde{m}_\alpha \geq 0.005$ eV and at least one ≥ 0.02 eV.

The lower bounds are stronger for inverted hierarchy, and even more so in the $3+3$ framework with quasi-degenerate light neutrinos.

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